# Graph Analysis: Edge Fixing and Its Impact on Detour Number Metrics

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## ABSTRACT

We introduce the concept of the total edge fixing edge-to-vertex detour setof a connected graph G. Let e be an edge of a graph G. A set $S(e) \subseteq E(G) - \{e\}$is called an edge fixing edge-to-vertex detour set of a connected graph G if every edge of G lies on an $e - f$ detour, where $f \in S(e)$. The edge fixing edge-to-vertex detour number defev(G) of G is the minimum cardinality of its edge fixing edge-to-vertex detour sets and any edge fixing edge-to-vertex detour set of cardinality $dn_{efev}(G)$ is an $d_{efev}$-set of G. Connected graphs of order p with edge fixing edge-to-vertex detour number 1 or $q - 1$ are characterized. Theedge fixing edge-to-vertex detour number for some standard graphs are determined. It is shown that for every pair of positive integers with $2 \leq a \leq b$, there exists a connected graph G such that $dn_{ev}(G) = a$ and $dn_{efev}(G) = b$, for some edge $e \in E(G)$..

**Keywords:** *detour set ,edge-to-vertex detour set , edge fixing edge –to-vertex detour set,edge fixing edge - to vertex detour set,edge fixing edge - to - vertex detour number.*
***Mathematical subject classification 05C12.***

## I. INTRODUCTION

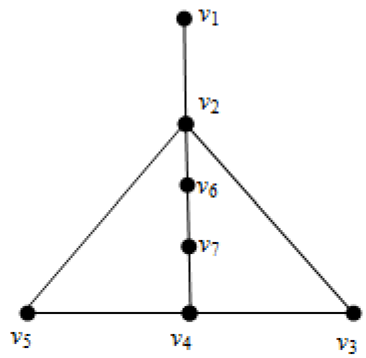
For a graph $G = (V, E)$, we mean a finite undirected graph without loops or multiple edges. The order and size of $G$ are denoted by $p$ and $q$ respectively. We consider connected graphs with at least two vertices. For basic definitions and terminologies we refer to [1,4].For vertices $u$ and $v$ in a connected graph $G$, the *detour distance*$D(u,v)$ is the length of the longest $u - v$ path in $G$.A $u - v$ path of length $D(u,v)$ is called a $u - v$*detour*. It is known that the detour distance is a metric on the vertex set $V(G)$. The *detour eccentricity*$e_D(v)$ of a vertex $v$ in $G$ is the maximum detour distance form $v$ to a vertex of $G$.The detour *radius*,$rad_D G$ of $G$is the minimum detour eccentricity among the vertices of $G$, while the *detour diameter*, $diam_D G$of $G$ is the maximum detour eccentricity among the vertices of $G$. These concept were studied by Chartrand et al.[2]. Let $G = (V, E)$ be a connected graph with at least 3 vertices. A set $S \subseteq E$ is called an *edge-to-vertex detour set* if every vertex of $G$ is either incident with an edge of $S$ or lies on a detour joining a pair of edges of $S$. The *edge-to-vertex detour number*$d_{ev}(G)$ of $G$ is the minimum cardinality of its edge-to-vertex detour sets and any edge-to-vertex detour set of cardinality $d_{ev}(G)$ is an *edge-to-vertex detour*$d_{ev}$*-set* of $G$.

**Theorem 1.1[6]**
Every pendant edge of a connected graph $G$ belongs to every edge-to-vertex detour set of $G$.

**Theorem 1.2[6]**
For any non-trivial tree $T$ with pendant edges, $d_{ev}(T) = k$ and the set of all pendant edges of $T$ is the unique minimum edge-to- vertex detour set of $T$.

## II. THE EDGE FIXING EDGE-TO-VERTEX DETOUR

**Number of a Graph**
**Definition 2.1**
Let $e$ be an edge of a graph $G$. A set $S(e) \subseteq E(G) - \{e\}$ is called an *edge fixing edge-to-vertex detour set* of a connected graph $G$ if every edge of $G$ lies on an $e - f$ detour, where $f \in S(e)$. The *edge fixing edge-to-vertex detour number* $d_{efev}(G)$ of $G$ is the minimum cardinality of its edge fixing edge-to-vertex detour sets and any edge fixing edge-to-vertex detour set of cardinality $d_{efev}(G)$ is an $d_{efev}$ -set of $G$.
**Example 2.2**
For the graph$G$ given in Figure 2.1, the edge fixing edge-to-vertex detour sets of each edge of $G$ is given in the following Table 2.1.




*G*
***Figure 2.1***

***Table 2.1***

| **Fixing Edge ($e$)** | **Minimum edge fixing edge-to-vertex detour sets ($S(e)$)** | $d_{efev}(S(e))$ |
|---|---|---|
| $v_1v_2$ | $\{v_2v_6\}$,$\{v_6v_7\}$ | 1 |
| $v_2v_3$ | $\{v_1v_2, v_6v_7\}$ | 2 |
| $v_3v_4$ | $\{v_1v_2, v_4v_5\}$ | 2 |
| $v_4v_5$ | $\{v_1v_2, v_3v_4\}$ | 2 |
| $v_2v_5$ | $\{v_1v_2, v_6v_7\}$ | 2 |
| $v_6v_2$ | $\{v_1v_2\}$ | 1 |

**Remark 2.3**

For a connected graph $G$, the edge $e$ of $G$ does not belong to the edge fixing edge-to- vertex detour set $S(e)$. Also the edge fixing edge-to- vertex detour set of an edge $e$ is not unique. For the graph $G$ given in Figure 6.1, the edge fixing edge-to- vertex detour sets of the edge $v_1v_2$are $\{v_6v_7\}$, $\{v_2v_6\}$.

## III. SOME RESULTS ON THE EDGE FIXING EDGE-TO-VERTEXDETOURNUMBER OF A GRAPH

**Theorem 2.4**

Let $e$ be an edge of $G$. Let $f$ be a pendant edge of a connected graph $G$such that $e \neq f$. Then every edge fixing edge-to- vertex detour set of $e$of $G$ contains $f$.

**Proof.** Since $e \neq f, f$ is a terminal edge of a detourhence $f$belongs to every edge fixing edge-to- vertex detour set of $e$of$G$. ■

**Theorem 2.5**

Let $G$ be a connected graph and $S(e)$ be an edge fixing edge-to- vertex detour set of $e$of $G$. Let $f$ be a non-pendant cut edge of $G$and let$G_1$ and $G_2$ be the two component of $G - \{f\}$. If $e = f$, then each of the two component of $G - \{f\}$ contains an element of $S(e)$. If $e \neq f$, then $S(e)$ contains at least one edge of component of $G - \{f\}$ where $e$ does not lie.

**Proof.** Let $f = uv$. Let $G_1$and $G_2$ be the two component of $G - \{f\}$ suchthat$u \in V(G_1)$and $\in V(G_2)$.Let $e = f$. Suppose that $S(e)$ does not contain any element of $G_1$. Then$S(e) \subseteq E(G_2)$.Let $h$ be an edge of $E(G_1)$. Then $h$ must lie on an $e$– $f'$detourforsome $f' \in S(e)$. But such a detour $P$: $v, v_1, v_2,\ldots,v_l, v, u, u_1, u_2,\ldots, u_s, u, v, v_1, v_2,\ldots, v'$ where

$v_1, v_2, \ldots, v_l \in V(G_2)$ , $u_1, u_2, \ldots, v_s \in V(G_1)$ and $v'$ is an end of $f'$ has the cut-edge $f$ twice, hence it is a contradiction. This proves the theorem.By similar argument, we can prove that if $e \neq f$, then $S(e)$ contains at least one edge from a component of $G - \{f\}$ where $e$ does not lie. ■

**Theorem 2.6**
Let $G$ be a connected graph and $S(e)$ be a minimum edge fixing edge-to- vertex detour set of an edge $e$of $G$. Then no non-pendant cut-edge of $G$ belongs to $S(e)$.

**Proof.**Let $S(e)$ be an edge fixing edge-to- vertex detour set of an edge $e = uv$of $G$. Let $f = u'v'$be a non-pendant cut-edge of $G$ such that $f \in S(e)$. Since $e \neq f$, let $G_1$ and $G_2$ be the two component of $G$ - $\{f\}$ such that $u' \in V(G_1)$ and $v' \in V(G_2)$. By Theorem6.5, $G_1$ contains an edge $xy$and $G_2$ contains an edge $x'y'$ where $xy, x'y' \in S(e)$. Let $S'(e) = S(e)$- $\{f\}$. We claim that $S'(e)$ is an edge fixing edge-to- vertex detour set of an edge $e$of $G$.

**Case 1.** Suppose that $e = xy$ is an edge in $G_1$ and $x'y'$is an edge in $G_2$. Let $h$ be avertexof $G$. Assume without loss of generality that $h = wz$belongs to $G_1$. Since $u'v'$is a cut-edge of $G$, every path joining an edge of $G_1$ with an edge of $G_2$ contains the edge $u'v'$. Suppose that $h$ is adjacent with $u'v'$or the edge $xy$ of $S(e)$ or that lies on a detour joining $xy$and $u'v'$. If $h$ is adjacent with $u'v'$,then $z = u'$. Let $P : x, y, y_1, y_2, \ldots, w,\ z = u'$beaxy $-u'v'$ detour. Let $Q: u', v', v_1', v_2', \ldots, x', y'$ a $u'v'$-$x'y'$detour. Then, it is clear that $P$ followed by $u'v'$and $Q$ is a $xy$-$x'y'$detour. Thus $h$ lies on the $xy$-$x'y'$ detour. If $h$is adjacent with $xy$, then there is nothing to prove. If $h$ lies on a $xy$-$x'y'$detour, say $x, y, v_1, v_2, \ldots, w, z, \ldots, u', v'$, then let $u', v', v_1', v_2', \ldots, y'$ be $u'v'$-$x'y'$detour. Thenclearly$x, y, v_1, v_2, \ldots, w, z, \ldots, u', v', v_1', v_2', \ldots, x', y'$is a $xy$-$x'y'$detour. Thus $h$ lies on a detour joining$xy$and an element of $S'(e)$. Thus we have proved that an edge that is adjacent with $u'v'$or an edge of $S(e)$ or that lies on a detour joining $xy$ and $u'v'$of $S(e)$ also is adjacent with an edge of $S'(e)$ or lies on a detour joining $e$ and an edge of $S'(e)$. Hence it follows that $S'(e)$ is an edge fixing edge-to- vertex detour set of an edge $e$ of $G$ such that $|S'(e)| = |S(e)|$ - 1, which is a contradiction to the minimality of $S(e)$.

**Case 2.**Suppose that $e = xy \in G_2$.The proof is similar to that of Case 1. Hence the theorem follows. ■

**Theorem 2.7**
For any non-trivial tree $T$ with $k$end edges,

$$d_{efev}(G) = \begin{cases} k-1 & \text{if } e \text{ is an end edge of } G \\ k & \text{if } e \text{ is an internal edge of } G \end{cases}.$$

**Proof.** This follows from Theorem 2.4 and Theorem 2.6. ■

**Theorem 2.8**
For the graph $G = C_p (p \geq 4), d_{efev}(G) = 1$, for any edge $e$ of $E(G)$.

**Proof.** Let $C_p: v_1, v_2, v_3, \ldots, v_p$ be the cycle. Let $e$be an edge of $C_p$and $f$ be an edge adjacent to $e$. Then it follows that $\{f\}$ is an edge fixing edge-to- vertex detour set of an edge $e$of $C_p$. Hence $d_{efev}(C_p) = 1$. ■

**Theorem 2.9**
For the complete graph $K_p (p \geq 4), d_{efev}(G) = 1$ for every edge in $E(G)$.
**Proof.** We observe that all the edges of $K_p$can be considered as the edges of $C_p$and every edge joining the points of $C_p$. Let $e$be an edge of $C_p$and $f$ be an edge adjacent to $e$. Then it follows that $\{f\}$ is an edge fixing edge-to-vertex detour set of an edge $e$ of $C_p$.Hence $d_{efev}(K_p) = 1$.■

**Theorem 2.10**
Let $G$ be a connected graph with at least three vertices. Then $1 \leq d_{efev}(G) \leq q - 1$.

**Proof.** For any edge $e$ in $G$, an edge fixing edge-to-vertex detour set needs at least one edge of $G$ so that $d_{efev}(G) \geq 1$. For an edge $e \in E(G)$, $E(G) - \{e\}$ is an edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G) \leq q - 1$. Therefore $1 \leq d_{efev}(G) \leq q - 1$. ■

**Remark 2.11**
The bounds in Theorem 2.10 are sharp. For the cycle $G = C_p$ $(p \geq 4)$, for an edge $e$, any edge which is adjacent to$e$ is its minimum edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G) = 1$. For the star $G = K_{1,q}$, for an edge $e$, the set of edges $E(G) - \{e\}$ is the unique edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G) = q - 1$ . Thus the star $K_{1,q}$ has the largest possible edge fixing edge-to-vertex detour number $q - 1$ and the cycle $G = C_p$ $(p \geq 4)$, has the smallest edge fixing edge-to-vertex detour number 1. Also the bounds in Theorem 2.10 is strict. For the graph $G$ given in Figure 2.1, for the edge $e = v_3v_4, d_{efev}(G) = 2$ so that $1 < d_{efev}(G) < q - 1$.

**Theorem 2.12**
Let $G$ be a connected graph of size $q \geq 3$, such that $G$ is neither a star nor a double star. Then $d_{efev}(G) \leq q - 2$ for every $e \in E(G)$.

**Proof.**
**Case 1.** Suppose that $G$ is a tree such that $G$is neither a star nor a double star. Then by Theorem 2.7,$d_{efev}(G) \leq q - 2$, for every $e \in E(G)$.

**Case 2.** Suppose that $G$ is not a tree. Then $G$ contains at least one cycle, say $C$. Let $e$ be an edge of $G$

**Subcase 2a.**Suppose that$e \in E(C)$. Then $S(e) = E(G) - E(C)$is an edge fixing edge-to-vertex detour set of an edge $e$ of $G$ so that $d_{efev}(G) \leq q - 2$.

**Subcase 2b.** Suppose that$e \notin E(C)$. Then setting $S(e) = E(G) - E(C) - \{e\}$ and by the similar argument in Subcase2a we can prove that $d_{efev}(G) \leq q - 2$. Hence the proof. ■

**Remark 2.13**
The bound in Theorem2.12 is sharp. For the graph $G = C_3$, it is easily verified that $d_{efev}(G) = q - 2$ for every edge $e$of $G$.

**Theorem 2.14**
Let $G$be a connected graph of size $q \geq 2$ and $e \in E(G)$. Then $d_{efev}(G) = q - 1$ if and only if $e$ is an edge of $K_{1,q}$ or $e$ is an internal edge of a double star.

**Proof.** Let $G$ be a connected graph. If $e$ is an edge of $K_{1,q}$, then by Theorem 2.7,$d_{efev}(G) = q - 1$. If $e$ is an internal edge of a double star, then by Theorem 2.7,$d_{efev}(G) = q - 1$ .
Conversely, let $d_{efev}(G) = q - 1$ for an edge $e \in E(G)$. Suppose that$e$is neither an edge of $K_{1,q}$nor an internal edge of a double star. Then by Theorem 2.12,$d_{efev}(G) = q - 2$ , which is a contradiction.Therefore$e$ is an edge of $K_{1,q}$ or $e$is an internal edge of a double star.■

**Theorem 2.15**
Let $G$ be a connected graph with $q \geq 4$, which is not a cycle and not a tree and let $C(G)$ be the length of the longest cycle. Then $d_{efev}(G) \leq q - C(G) + 1$ for some $e \in E(G)$.

**Proof.** Let $C(G)$ denote the length of the longest cycle in $G$ and $C$ be the cycle of length $k$.
Let $C: v_1, v_2, v_3, \ldots, v_k$ be a cycle, $k \geq 3$. Since $G$ is not a cycle, there exists a vertex $v$in $G$ such that $v$ is not a vertex of $C$ and which is adjacent to $v_1$, say. Let $e$ be an edge of $C$. Let $S(e) = E(G) - \{E(C) - e\}$. Clearly $S(e)$ is an edge fixing edge-to-vertex detour set of $e$of$G$ so that $d_{efev}(G) \leq q - C(G) + 1$ . ■

**Theorem 2.16**
Let $G$ be a connected graph of size $q \geq 3$ which is not a double star and $d_{efev}(G) = q - 2$for some edge $e$of $G$. Then $G$ is unicyclic.

**Proof.** Suppose that $G$ is not unicyclic. Then $G$ contains more than one cycle.

Let $C_1$ and $C_2$ be the two cycles of $G$. By Theorem 2.15, $|C_1| = |C_2| = 3$.

**Case 1.** Suppose that $C_1$ and $C_2$have exactly one vertex, say, $v$in common.
Let $e = uv$ be an edge of $C_1$and let $S(e) = E(G) - E(C) - \{e, f\}$, where $f = vw$, where $w \in V(C_2)$. Then $S(e)$ is an edge fixing edge-to-vertex detour set of an edge $e$ of $G$ so that $d_{efev}(G) = q - 3$, which is a contradiction.

**Case 2.**Suppose that $C_1$ and $C_2$have a common edge, say, $uv$.
Let $e = uv$ and let $S(e) = E(G) - \{e, uw, uz\}$, where $w \in V(C_1)$ and $z \in V(C_2)$. Then $S(e)$ is an edge fixing edge-to-vertex detour set of $e$of$G$ so that $d_{efev}(G) = q - 3$, which is a contradiction.

**Case 3.**Suppose that $C_1$ and $C_2$ are connected by a path $P$.
Suppose that $e = xu$ be an edge of $C_1$, where $x$ is a vertex common to $C_1$ are $P$ and let $S(e) = E(G) - \{e, xu_1, xx_1, f\}$,where $xu_1 \in E(C_1)$ such that $u \neq u_1, xx_1 \in E(P)$ and $f \in E(C_2)$. Then clearly $S(e)$ is an edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G) \leq q - 4$, which is a contradiction. ■

**Theorem 2.17**
For a connected graph $G$, $d_{ev}(G) \leq d_{efev}(G) + 1$.

**Proof.** Let $e$ be an edge of $G$ and $S(e)$ be the minimum edge fixing edge-to-vertex detour set of $e$of $G$. Then $S(e) \cup \{e\}$ is an edge-to-vertex detour set of $e$ of $G$ so that $d_{ev}(G) \leq |S(e) \cup \{e\}| = d_{efev}(G) + 1$. ■

**Remark 2. 18**
The bound in Theorem 2.17 is sharp. For the cycle $C_p$, $d_{efev}(C_p) = 1$for every $e \in E(G)$ and $d_{ev}(G) = 2$ so that $d_{ev}(G) = d_{efev}(G) + 1$.Also the inequality in the Theorem 2.17 strict. For the graph $G$given in Figure 2.2, let $e = u_3u_4$. Then $S(e) = \{u_1u_2, u_7, u_8\}$ is an edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G) = 2$. Also $d_{ev}(G) = 2$. Hence $d_{ev}(G) < d_{efev}(G) + 1$.

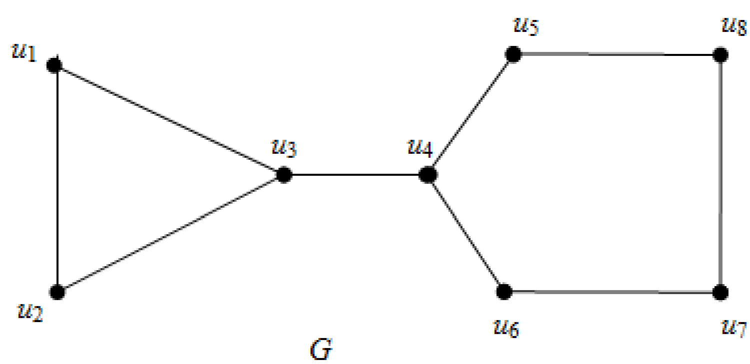



*Figure 2.2*

**Theorem 2.19**
For positive integers $R, D$and $l \geq 2$with $R < D \leq 2R$, there exists a connected graph $G$ with $rad(G) = R, diam(G) = D$ and $d_{efev}(G) = l$ for some $e \in E(G)$.

**Proof.**When $R = 1$, we let $G = K_{1,l}$. Then the result follows from Theorem 2.7. Let $R \geq 2$. Let $C_{R+1}: v_1, v_2, \dots, v_{R+1}$ be a cycle of length $R + 1$ and let $P_{D-R}: u_0, u_1, u_2, \dots, u_{D-R}$ be a path of length $D - R$. Let $H$ be a graph obtained from $C_{R+1}$ and $P_{D-R}$ by identifying $v_1$ in $C_{R+1}$ and $u_0$in $P_{D-R}$. Now add $l - 2$ new vertices $w_1, w_2, \dots, w_{l-2}$ to $H$ and join each $w_i$ $(1 \leq i < l - 2)$ to the vertex $u_{D-R\_1}$ and obtain the graph $G$ as shown in Figure 2.3. Then $rad_D(G) = R$and $diam_D(G) = D$. Let $S = \{u_{D-R-1}u_{D-R}, u_{D-R-1}w_1, u_{D-R-1}w_2, \dots, u_{D-R-1}w_{l-2}\}$ be the set of end-edges of $G$. Let $e$be a non-pendant cut edge of $G$.By Theorem 2.4, $S$ is a subset of every edge fixing edge-to-vertex detour set of $G$. It is clear that $S$ is not an edge fixing edge-to-vertex detour set of $G$ and so $d_{efev}(G) \geq l$. However $S \cup \{v_1v_2\}$ is an edge fixing edge-to-vertex detour set of $e$ of $G$ and so that $d_{efev}(G) = l$. ■

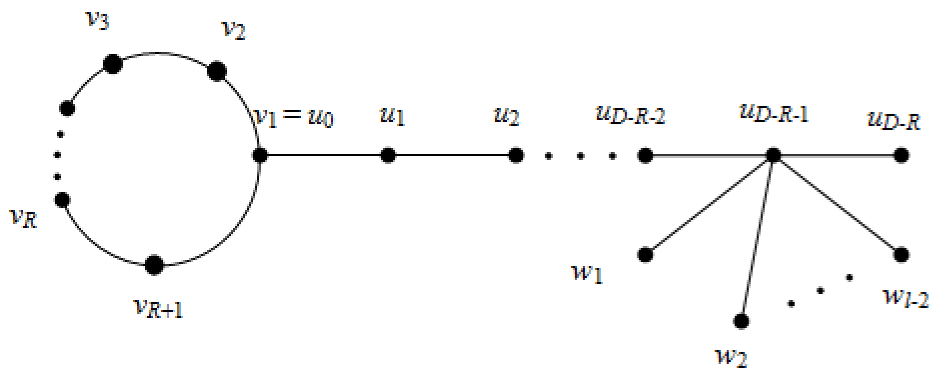



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***Figure 2.3***

**Theorem 2.20**
For any positive integer $a, 1 \leq a \leq q - 1$, there exists a connected graph $G$ of size $q$ such that $d_{efev}(G) = a$, for some edge $e \in E(G)$.

**Proof.** Let $G$ be a connected graph.

**Case 1.**Let $a = q - 1$.
For the star $G = K_{1,q}$, by Theorem 6.7,$d_{efev}(G) = q - 1 = a$ for every edge $e \in E(G)$.

Case 2.$a = 1$
Let $G$ be a path of length $q$ and $e$ be an pendant-edge of $G$. Then by Theorem 2.7, $d_{efev}(G) = 1 = a$.

**Case 3.**$1 < a < q - 1$
Let $G$ be a tree with $a$ end-edges and $q - a$ internal edges and let $e$ be an internal edge of $G$. Then by Theorem 2.7, $d_{efev}(G) = a$. ■
In view of Theorem 2.17, we have the following realization result.

**Theorem 2.21**
For every pair of positive integers with $2 \leq a \leq b$, there exists a connected graph $G$ such that $d_{ev}(G) = a$ and $d_{efev}(G) = b$ for some edge $e \in E(G)$.

**Proof.** Let $G$ be a connected graph.

**Case 1.**$a = b$
Let $G$ be a double star with $a$ end-edges and let $e$ be the cut-edge of $G$. Then by Theorem 2.8, $d_{efev}(G) = a$. Also by Theorem 1.2, $d_{ev}(G) = a$.

**Case 2.**$2 \leq a < b$
Let $P : u_1, u_2, u_3, u_4, u_5, u_6, u_7$,be a path of order 7. Let $P_i : x_i y_i (1 \leq i \leq b - a + 1)$ be a copy of a path of order 2. Let $H$ be a graph obtained from the path on $P$and$P_i$ by joining $u_1$ with each $x_i (1 \leq i \leq b - a + 1)$ and $u_7$with $y_i (1 \leq i \leq b - a + 1)$. Let $G$ be the graph obtained from $H$ by adding new vertices $z_1, z_2, \ldots, z_{a-1}$ and joining each $z_i (1 \leq i \leq a - 1)$with $u_7$. The graph $G$ is shown in Figure 2.4. First show that $d_{ev}(G) = a$. Let $S = \{z_1 u_7, z_2 u_7, \ldots, z_{a-1} u_7\}$ be the set of all pendant-edges of $G$. By Theorem 1.1, $S$ is a subset of every edge-t0-vertexdetour set of $e$of$G$. It is clear that $S$ is not an edge-to-vertex detour set of $G$ and so $d_{ev}(G) \geq a - 1$.However $S' = S \cup \{u_6 u_7\}$ is an edge-to-vertex detour set of $G$. Thus $d_{ev}(G) = a$. Let $e = u_1 x_1$. By Theorem2.4, $S = \{z_1 u_7, z_2 u_7, \ldots, z_{a-1} u_7\}$ is a subset of every edge fixing edge-to-vertex detour set of $e$ of $G$. It is clear that $S$is not an edge fixing edge-to-vertex detour set of $e$of $G$. It is easily verified that every edge fixing edge-to-vertex detour set

of $e$ of $G$contains $x_i y_i$ $(2 \leq i \leq b - a + 1)$ and so $d_{efev}(G) \geq a - 1 + b - a + 1 = b$. Let $S(e) = S \cup \{x_1y_1, x_2y_2, \dots, x_{b-a+1}, y_{b-a+1}\}$. Then$S(e)$ is an edge fixing edge-to-vertex detour set of $e$ of $G$ so that $d_{efev}(G)= b$. Hence the proof. ■

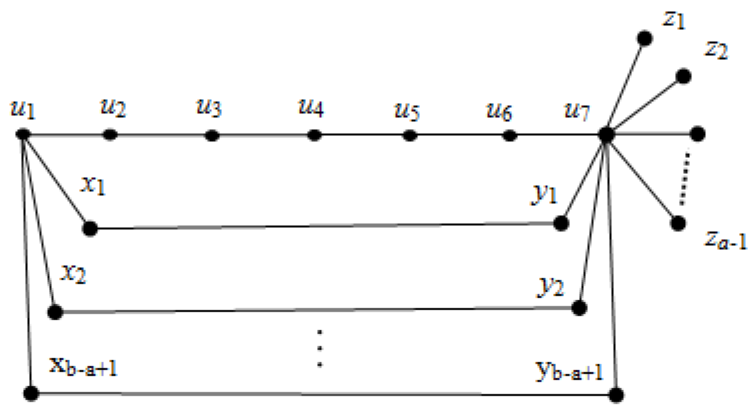


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***Figure 2.4***


## REFERENCES

1. *F.Buckley and F.Harary, Distance in Graph,Addition-Wesley, reading MA,1990.*
2. *G.Chartrand, H.Escuadro and B.Zang. The Detour Distance in Graphs, J.Combin.math.combin.compute.,53(2005),75-94.*
3. *G.Chartrand,N.Johns,andP.Zang, The Detour number of Graph,Util.math.64(2003),97-113.*
4. *Haynes T W, Hedetniemi S .T and Slater P. J, Fundamentals of Domination in graphs, Marcel Decker ,Inc., New York 1998.*
5. *G.Chartrand,andP.Zang , Introduction to Graph Theory, TataMcGraw-Hill-New Delhi,(2006).*
6. *A. P. Santhakumaran and S. Athisayanathan, The Edge-to-vertex detour number of a graph, Adv. Studies Contem. Math., 21 (2011), No. 4, 395-412*