ON FUZZY SUPRA ALMOST P-SPACES

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Abstract.

The concept of Almost P-spaces in classical topology was introduced by A.I.Veksler [16] as P'-spaces and was also studied further by R.Levy [7] and C.I. Kim [6]. The aim of this paper is to introduced and studied the notion of fuzzy supra almost P-spaces and various characterizations of fuzzy supra almost P-spaces are established.

Keywords:

Fuzzy supra F_σ-set, Fuzzy supra G_δ-set, Fuzzy supra nowhere dense set, Fuzzy supra dense set, Fuzzy supra P-Space, Fuzzy supra almost P-Space.

1. Introduction

In the year 1965, L. A.Zadeh [17] was first introduced the concept of fuzzy sets and fuzzy set operations in his classical paper. In 1968, the theory of fuzzy topological spaces was introduced and developed by C.L.Chang [4]. In 1983, Mashhour.A.S.et.al.,[8] introduced and studied the concept of Supra topological spaces. In 1987, Abd El-monsef et.at [1] introduced the concepts of fuzzy supra topological as a natural generalization of the notion of supra topological spaces. In 1972, Mishra.A.K [9] introduced the concepts of P-spaces as a generalization of ω_α-additive spaces of Sikorski [13] and Cohen.L.W and Goffman.C [5]. The concept of P-spaces in fuzzy setting was introduced by Thangaraj.G and Balasubramanian.G [15]. The concept of fuzzy supra P-spaces and weak fuzzy supra P-spaces was introduced and studied by the author in [11,12].

In this paper, fuzzy supra almost P-spaces is introduced and studied. It is established that fuzzy supra G_δ-sets and fuzzy supra residual sets in fuzzy supra almost P-spaces are fuzzy supra somewhere dense sets. It is obtained that fuzzy supra σ-boundary sets in fuzzy supra almost P-spaces are either fuzzy supra first category sets which are not dense or fuzzy supra somewhere dense sets.

2. Preliminaries

Definition 2.1 [1].

A collection S* of fuzzy sets in a set M is called fuzzy supra topology on M if the following conditions are satisfied:

1) **0** and **1** belongs to S*.

2) $g_\chi \in S^*$ for each $\chi \in \Lambda$ implies $(\vee_{\chi \in \Lambda} g_\chi) \in S^*$.

The pair (M,S*) is called a fuzzy supra topological space. The elements of S* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition: 2.2 [10]

Let (M,S*) be a fuzzy supra topological space and B be a fuzzy set in M, then the fuzzy supra closure and fuzzy supra interior of B defined respectively as

cl*(B) = ∧ { g / g is a fuzzy supra closed set in U and B ≤ g}

int*(B) = v { g / g is a fuzzy supra open set in U and g ≤ B}

Definition: 2.3 [10]

Let (U,T) be a fuzzy topological space and S* be a fuzzy supra topology on U. We call S* a fuzzy supra topology associated with T if T ≤ S*.

Remark: 2.4 [1]

(1). Every fuzzy topological space is a fuzzy supra topological space.

(2). If (M,S*) is an associated fuzzy supra topological space with the fuzzy topological space (M,S), then every fuzzy open(closed) set in the fuzzy topological space (M,S) is fuzzy supra open(closed) set in the fuzzy supra topological space (M,S*).

Lemma 2.5 [4]

For a fuzzy set B in a fuzzy topological space M,

(i) 1 – int*(B) = cl*(1– B),

(ii) 1 – cl*(B) = int*(1 – B).

Definition 2.6 [3]

A fuzzy supra open set B in fuzzy supra topological space (M,S*) is called fuzzy supra F_σ-set in (M,S*) if B = $\vee_{i=1}^{\infty}(B_i)$, where $1 – B_i \in S^*$ for i∊ I,

Definition 2.7 [3]

A fuzzy supra open set B in fuzzy supra topological space (M,S*) is called fuzzy supra G_δ -set in (M,S*) if B =$\wedge_{i=1}^{\infty}(B_i)$, where $B_i \in S^*$ for i∊ I.

Definition 2.8 [3]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra dense set if there exists no fuzzy supra closed set D in (M,S*) such that B<D<1. That is, cl*(B)=1, in (M,S*).

Definition 2.9 [3]

A fuzzy set B in fuzzy supra topological space (M,S*) is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set A in (M,S*) such that A < cl*(B). That is, int*cl*(B)=0, in (M,S*).

Definition 2.10 [3]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra first category set if B = $\vee_{i=1}^{\infty}(B_i)$, where (B_i)'s are fuzzy supra nowhere dense set in (M,S*). Any other fuzzy set in (M,S*) is said to be fuzzy supra second category space.

Definition 2.11 [2]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra boundary of B is defined as Bd(B) = cl*(B)∧cl*(1−B). Obviously Bd(B) is a fuzzy supra closed set in (M,S*).

Definition 2.12 [3]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra simply-open set if Bd(B) is a fuzzy supra no-where dense set in (M,S*). That is., B is an fuzzy supra simply-open set in (M,S*) if int*cl*[cl*(B) ∧ cl*(1- B)] = 0 in (M,S*).

Definition 2.13 [2]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra semi-open set in (M,S*) if B ≤ cl*int*(B); fuzzy supra semi-closed set in (M,S*) if int*cl*(B) ≤ B.

Definition 2.14 [3]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra somewhere dense set if there exists a non-zero fuzzy open set B in (M,S*) such that B < cl*(B). That is., int*cl*(B) ≠ 0, in (M,S*).

Definition 2.15. [13]

A fuzzy set B in a fuzzy supra topological space (M,S*) is called a fuzzy supra regular-open set if B = int*cl*(B) and fuzzy supra regular closed set if B = cl*int*(B) in (M,S*).

Theorem 2.1. [3]

In the fuzzy supra topological space

(a) The closure of a fuzzy supra open set is a fuzzy supra regular closed set.

(b) The interior of a fuzzy supra closed set is a fuzzy supra regular open set.

Theorem 2.2. [3]

If A is a fuzzy supra σ-boundary set in a fuzzy supra topological space (M,S*), then A is a fuzzy supra F_σ-set in (M,S*).

Theorem 2.3. [3]

If A is a fuzzy supra co-σ-boundary set in (M,S*), then 1-A is a fuzzy supra σ-boundary set in (M,S*).

Theorem 2.4. [3]

If A is a fuzzy supra dense and fuzzy supra G_δ-set in a fuzzy supra topological space (M,S*), then A is a fuzzy supra residual set in (M,S*).

Theorem 2.5. [3]

If A is a fuzzy supra somewhere dense set in a fuzzy supra hyperconnected space (M,S*), then cl*(A)∨cl*(1-A) = 1 in (M,S*).

Theorem 2.6. [3]

For any fuzzy supra topological space (M,S*), the following are equivalent:

(a) (M,S*) is fuzzy supra basically disconnected space.

(b) For each fuzzy supra closed G_δ-set A, int*(A) is fuzzy supra closed.

(c) For each fuzzy supra open set F_σ-set A, cl*(A) + cl*[1-cl*(A)] = 1.

Theorem 2.7. [3]

Let (M,S*) be a fuzzy supra topological space. Then, the following are equivalent:

(a) (M,S*) is fuzzy supra hyperconnected.

(b) Every fuzzy supra pre-open set is fuzzy supra dense set.

Theorem 2.8. [3]

If A is a fuzzy supra set defined on M with int*(A) ≠ 0 in a fuzzy supra hyperconnected space (M,S*), then int*cl*(A) and 1-int*cl*(A) are fuzzy supra resolvable sets in (M,S*).

Theorem 2.9. [3]

If A is a fuzzy supra residual set in a fuzzy supra globally disconnected space (M,S*), then A is a fuzzy supra G_δ-set in (M,S*).

Theorem 2.10. [3]

Let (M,S*) be a fuzzy supra topological space. Then, the following properties are equivalent:

(i) (M,S*) is fuzzy supra hyperconnected.

(ii) 1_M and 0_M are the only fuzzy supra regular open sets in M.

Theorem 2.11. [3]

If a fuzzy supra topological space (M,S*) is a fuzzy supra σ-Baire space, then (M,S*) is a fuzzy supra Volterra space.

Theorem 2.12. [3]

Let (M,S*) be a fuzzy supra topological space. Then, the following are equivalent:

(i) (M,S*) is a fuzzy supra weakly Baire space.

(ii) int*(A) = 0, for every fuzzy supra σ-boundary set A in (M,S*).

(iii) cl*(B) = 1, for every fuzzy supra co-σ-boundary set B in (M,S*).

Theorem 2.13. [3]

If a fuzzy supra topological space (M,S*) is a fuzzy supra weakly Baire space and fuzzy supra open hereditarily irresolvable space, then (M,S*) is a fuzzy supra Baire space.

Theorem 2.14. [3]

If a fuzzy supra topological space (M,S*) is a fuzzy supra weakly Baire space, then for any regular open set A in (M,S*), A∧int*(1-A) = 0, in (M,S*).

3.Fuzzy Supra Almost P-Spaces

Definition 3.1.

A fuzzy supra topological space (M,S*) is called a fuzzy supra almost P-space, if for every non-zero fuzzy supra G_δ-set A in (M,S*), $int^*(A) \neq 0$ in (M,S*).

Proposition 3.1.

If A is a fuzzy supra G_δ-set in a fuzzy supra almost P-space (M,S*), then A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A) \neq 0$, in(M,S*). Now $int^*(A) \leq int^*cl^*(A)$, implies that $int^*cl^*(A) \neq 0$, in (M,S*). Hence, A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 3.2.

If A is a fuzzy supra G_δ-set in a fuzzy supra almost P-space (M,S*), then $cl^*int^*(A)$ is a fuzzy supra regular closed set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A) \neq 0$, in (M,S*). By Theorem 2.1, for the fuzzy supra open set $int^*(A)$, $cl^*int^*(A)$ is a fuzzy supra regular closed set in (M,S*).

Proposition 3.3.

If B is a fuzzy supra F_σ-set in a fuzzy supra almost P-space (M,S*),then $int^*cl^*(A)$ is a fuzzy supra regular open set in (M,S*).

Proof.

Let A be a fuzzy supra F_σ-set in (M,S*). Then,1-A is a fuzzy supra G_δ-set in (M,S*).Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.2, $cl^*int^*(1-A)$ is a fuzzy supra regular closed set in (M,S*). Now $cl^*int^*(1-A)=1-int^*cl^*(A)$, in (M,S*). Then $1-int^*cl^*(A)$ is a fuzzy supra regular closed set in (M,S*) and hence $int^*cl^*(A)$ is a fuzzy supra regular open set in (M,S*).

Proposition 3.4.

If A is a fuzzy supra σ-boundary set in a fuzzy supra almost P-space (M,S*),then A is not a fuzzy supra dense set in (M,S*).

Proof.

Let A be a fuzzy supra σ-boundary set in (M,S*). By Theorem 2.2, A is a fuzzy supra F_σ-set in (M,S*).Then,1-A is a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(1-A)\neq 0$, in (M,S*). Then, $1-cl^*(A)\neq 0$ and $cl^*(A)\neq 1$. Hence A is not a fuzzy supra dense set in (M,S*).

Proposition 3.5.

If A is a fuzzy supra co-σ-boundary set in a fuzzy supra almost P-space (M,S*), then $int^*(A)\neq 0$ in (M,S*).

Proof.

Let A be a fuzzy supra co-σ-boundary set in (M,S*). Then, by Theorem 2.3, 1- A is a fuzzy supra σ-boundary set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.4, $cl^*(1-A)\neq 1$, in (M,S*). Then, $1-int^*(A)\neq 1$ and hence $int^*(A)\neq 0$ in (M,S*).

Proposition 3.6.

If A is a fuzzy supra G_δ-set in a fuzzy supra almost P-space (M,S*), then there exists a fuzzy supra regular closed set C in (M,S*) such that $C\leq cl^*(A)$.

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.1, A is a fuzzy supra somewhere dense set in (M,S*) and $int^*cl^*(A)\neq 0$. Then, there exists a non-zero fuzzy supra open set B in (M,S*) such that $B\leq cl^*(A)$. Now $cl^*(B) \leq cl^*[cl^*(A)]=cl^*(A)$. Since B is a fuzzy supra open set, by Theorem 2.1, $cl^*(B)$ is a fuzzy supra regular closed set in (M,S*). Let $C=cl^*(B)$. Thus there exists a fuzzy supra regular closed set C in (M,S*) such that $C\leq cl^*(A)$.

Proposition 3.7.

If A is a fuzzy supra G_δ-set in a fuzzy supra almost P-space (M,S*), then either A is a fuzzy supra residual set with $int^*(A) \neq 0$ or 1-A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Then, either $cl^*(A) =1$ or $cl^*(A)\neq 1$ in (M,S*). If $cl^*(A)=1$, in (M,S*), then A is a fuzzy supra dense and fuzzy supra G_δ-set in (M,S*). By Theorem 2.4, A is a fuzzy supra residual set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A)\neq 0$, in (M,S*).Thus, A is a fuzzy supra residual set with $int^*(A)\neq 0$, in (M,S*). If $cl^*(A)\neq 1$, in (M,S*), then $cl^*int^*(A)\leq cl^*(A)$, implies that $cl^*int^*(A)<1$ and then $1-cl^*int^*(A)>0$.Thus, $int^*cl^*(1-A)>0$.That is., $int^*cl^*(1-A) \neq 0$, in (M,S*). Hence, 1-A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 3.8.

If A is a fuzzy supra σ-boundary set in a fuzzy supra almost P-space (M,S*), then A is a fuzzy supra first category set with $cl^*(A)\neq 1$ or A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra σ-boundary set in (M,S*). By Theorem 2.2, A is a fuzzy supra F_σ-set in (M,S*). Then, 1-A is a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.7, either 1-A is a fuzzy supra residual set with $int^*(1-A)\neq 0$ or 1-(1-A) is a fuzzy supra somewhere dense set in (M,S*). Thus, A is a fuzzy supra first category set with $cl^*(A)\neq 1$ or A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 3.9.

If A is a fuzzy supra co-σ-boundary set in a fuzzy supra almost P-space (M,S*), then either A is a fuzzy supra residual set with $int^*(A) \neq 0$ or 1-A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra co-σ-boundary set in (M,S*). Then, by Theorem 2.3, 1-A is a fuzzy supra σ-boundary set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.8, 1-A is a fuzzy supra first category set with $cl^*(1\text{-}A) \neq 1$ or 1-A is a fuzzy supra somewhere dense set in (M,S*). Hence A is a fuzzy supra residual set with $int^*(A)\neq 0$ or 1 -A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 3.10.

If A is a fuzzy supra residual set in a fuzzy supra almost P-spaces (M,S*), then $int^*(A)\neq 0$ in (M,S*).

Proof.

Let A be a fuzzy supra residual set in (M,S*). Then, 1-A is a fuzzy supra first category set in (M,S*) and then $1\text{-}A= \vee_{i=1}^{\infty}(B_i)$, where (B_i)'s are fuzzy supra nowhere dense sets in (M,S*). Let $C=\wedge_{i=1}^{\infty}[1\text{-}cl^*(B_i)]$. Now $1\text{-}cl^*(B_i)$ is a fuzzy supra open set in (M,S*) and $C=\wedge_{i=1}^{\infty}[1\text{-}cl^*(B_i)]$ is a fuzzy supra G_δ-set in (M,S*). Also $C=\wedge_{i=1}^{\infty}[1\text{-}cl^*(B_i)]=1\text{-}[\vee_{i=1}^{\infty}cl^*(B_i)]\leq 1\text{-}[\vee_{i=1}^{\infty}(B_i)]=1\text{-}(1\text{-}A)$ and then $C \leq A$ in (M,S*). Then $int^*(C)\leq int^*(A)$ in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(C) \neq 0$ in (M,S*). Hence $int^*(A) \neq 0$ in (M,S*).

Proposition 3.11.

If A is a fuzzy supra residual set in fuzzy supra almost P-space (M,S*) then A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra residual set in (M,S*). Since (M,S*) is a fuzzy supra almost P-spaces, by Proposition 3.10, $int^*(A)\neq 0$ in (M,S*). Now $int^*(A)\leq int^*cl^*(A)$, implies that $int^*cl^*(A)\neq 0$, in (M,S*). Hence A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 3.12.

If A is a fuzzy supra G_δ-set in a fuzzy supra almost P-space (M,S*), then $int^*(A)$ is a fuzzy supra G_δ-set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Then, $A=\wedge_{i=1}^{\infty}(A_i)$, where $A_i \in S^*$. Since (M,S*) is a fuzzy supra almost P-space, $int^*(A)\neq 0$, in (M,S*) and $int^*(A)$ is a fuzzy supra open set in (M,S*). Now $int^*(A)\leq A$, $int^*(A)=int^*(A)\wedge A=int^*(A)\wedge[\wedge_{i=1}^{\infty}(A_i)]$ and thus $int^*(A)$ is a fuzzy supra G_δ-set in (M,S*).

4. Fuzzy Supra Almost P-Spaces and Other Fuzzy Supra Topological Spaces

Proposition 4.1.

If A is a fuzzy supra G_δ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then A is a fuzzy supra dense set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A) \neq 0$, in (M,S*). Now $int^*(A)$ is a fuzzy supra open set in the fuzzy supra hyperconnected space (M,S*) and thus is a fuzzy supra dense set in (M,S*). Then, $cl^* int^*(A)=1$, in (M,S*). Now $cl^*int^*(A) \leq cl^*(A)$ implies that $1 \leq cl^*(A)$. That is., $cl^*(A)=1$, in (M,S*). Hence A is a fuzzy supra dense set in (M,S*).

Remark 4.1.

In view of the above Proposition one will have the following result:" Fuzzy supra G_δ-sets in fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra dense sets".

Proposition 4.2.

If A is a fuzzy supra G_δ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (X, T), then

(i) $A \vee (1-A)$ is a fuzzy supra dense set in (M,S*).

(ii) $cl^*(A) \vee cl^*(1-A)=1$, in (M,S*).

Proof.(i).

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.1, A is a fuzzy supra dense set in (M,S*). Then, $cl^*(A)=1$, in (M,S*). Now $A \leq A \vee (1-A)$ implies that $cl^*(A) \leq cl^*(A \vee (1-A))$ and then $1 \leq cl^*(A \vee (1-A))$. That is., $cl^*(A \vee (1-A))=1$ and hence $A \vee (1-A)$ is a fuzzy supra dense set in (M,S*).

(ii). By Proposition 3.1, the fuzzy supra G_δ-set A is a fuzzy supra somewhere dense set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected space, by Theorem 2.5, $cl^*(A) \vee cl^*(1 - A) = 1$ in (M,S*).

Proposition 4.3.

If A is a fuzzy supra G_δ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then A is a fuzzy supra semi-open set in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A) \neq 0$, in (M,S*). Now $int^*(A)$ is a fuzzy supra open set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected space, $int^*(A)$ is a fuzzy supra dense set in (M,S*).Then, $cl^*int^*(A)=1$ and thus $A \leq cl^*int^*(A)$, in (M,S*). Hence A is a fuzzy supra semi-open set in (M,S*).

The following proposition shows that fuzzy supra basically disconnected and fuzzy supra hyperconnected spaces are not fuzzy supra almost P-spaces.

Proposition 4.4.

If (M,S*) is a fuzzy supra basically disconnected and fuzzy supra hyperconnected space, then (M,S*) is not a fuzzy supra almost P-space.

Proof.

Let A be a fuzzy supra closed G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra basically disconnected space, by Theorem 2.6, $int^*(A)$ is a fuzzy supra closed set in (M,S*). Then, $cl^*int^*(A)=int^*(A)$, in (M,S*). This implies that $cl^*int^*(A)\leq A$ and then A is a fuzzy supra pre-closed set in (M,S*). This implies that $1-A$ is a fuzzy supra pre-open set in (M,S*). Also since (M,S*) is a fuzzy supra hyperconnected space, by Theorem 2.7, $cl^*(1-A)=1$, in (M,S*). Then, $1-int^*(A) =cl^*(1-A) =1$ and $int^*(A) =0$, in (M,S*). Hence (M,S*) is not a fuzzy supra almost P-space.

Proposition 4.5.

If A is a fuzzy supra G_δ-set in the fuzzy supra hyperconnected and fuzzy supra almost P-space, then $int^*cl^*(A)$ and $1 - int^*cl^*(A)$ are fuzzy supra resolvable sets in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A)\neq 0$ in (M,S*). Also since (M,S*) is a fuzzy supra hyperconnected space, by Theorem 2.8, $int^*cl^*(A)$ and $1-int^*cl^*(A)$ are fuzzy supra resolvable sets in (M,S*).

Proposition 4.6.

If A is a fuzzy supra residual set in a fuzzy supra globally disconnected and fuzzy supra almost P-space (M,S*), then A is a fuzzy supra somewhere dense set in (M,S*).

Proof.

Let A be a fuzzy supra residual set in (M,S*). Since (M,S*) is a fuzzy supra globally disconnected space, by Theorem 2.9, A is a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, $int^*(A)\neq 0$, in (M,S*). Now $int^*(A)\leq int^*cl^*(A)$, implies that $int^*cl^*(A)\neq 0$ and hence A is a fuzzy supra somewhere dense set in (M,S*).

Proposition 4.7.

If A is a fuzzy supra residual set in a fuzzy supra globally disconnected and fuzzy supra almost P-space (M,S*), then there exists a fuzzy supra regular closed set C in (M,S*) such that $C \leq cl^*(A)$.

Proof.

Let A be a fuzzy supra residual set in (M,S*). Since (M,S*) is a fuzzy supra globally disconnected space, by Theorem 2.9, A is a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.6, there exists a fuzzy supra regular closed set C in (M,S*) such that $C\leq cl^*(A)$.

A condition for fuzzy supra door spaces to become fuzzy supra almost P-spaces, is obtained by means of somewhere denseness of fuzzy supra closed sets.

Proposition 4.8.

If fuzzy supra closed sets are fuzzy supra somewhere dense sets in a fuzzy supra door space (M,S*), then (M,S*) is a fuzzy supra almost P-space.

Proof.

Let (A_i)'s ($i=1$ *to* ∞) be fuzzy supra sets defined on M in (M,S*). Since (M,S*) is a fuzzy supra door space, (A_i)'s are either fuzzy supra open or fuzzy supra closed in (M,S*). Now consider those fuzzy supra sets (A_i)'s which are fuzzy supra open in (M,S*).

Let $B=\wedge_{i=1}^{\infty}(A_\alpha)$ where (A_α)'s are the fuzzy supra open sets from (A_i)'s in (M,S*). Then, B is a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra door space, the fuzzy supra set B is either fuzzy supra open or fuzzy supra closed in (M,S*). If B is a fuzzy supra open set in (M,S*), then $int^*(B)= B\neq 0$. If B is a fuzzy supra closed set in (M,S*), then $cl^*(B) = B$ and $int^*cl^*(B) = int^*(B)$. By hypothesis, fuzzy supra closed sets are fuzzy supra somewhere dense sets in (M,S*) and then $int^*cl^*(B)\neq 0$, in (M,S*). This implies that $int^*(B)\neq 0$, in (M,S*). Hence in both cases, $int^*(B)\neq 0$, for the fuzzy supra G_δ-set B in (M,S*). Hence (M,S*) is a fuzzy supra almost P-space.

Proposition 4.9.

If $B(\neq 1)$ is a fuzzy supra F_σ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then B is a fuzzy supra σ-nowhere dense set in (M,S*).

Proof.

Let $B(\neq 1)$ is a fuzzy supra F_σ-set in (M,S*). Since (M,S*) is a fuzzy supra almost P-space, by Proposition 3.3, $int^*cl^*(B)$ is a fuzzy supra regular open set in (M,S*). Also since (M,S*) is a fuzzy supra hyperconnected space, by Theorem 2.10, 1_X and 0_X are the only fuzzy supra regular open sets in (M,S*). Then, $int^*cl^*(B)=0$ or $int^*cl^*(B)=1$, in (M,S*). Now $int^*cl^*(B)=1$ implies that $cl^*(B)=0$ and then B is a fuzzy supra F_σ-set in (M,S*) such that $cl^*(B)=1$. But this is a contradiction to (M,S*) being a fuzzy supra almost P-space. Thus, $int^*cl^*(B)\neq 1$, in (M,S*). Then, $int^*cl^*(B)=0$ and $int^*(B)\leq int^*cl^*(B)$, implies that $int^*(B)= 0$, in (M,S*). Thus, B is a fuzzy supra F_σ-set in (M,S*) such that $int^*(B)=0$ in (M,S*) and hence B is a fuzzy supra σ-nowhere dense set in (M,S*).

The following proposition shows that fuzzy supra F_σ-sets in fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra first category sets.

Proposition 4.10.

If $A(\neq 1)$ is a fuzzy supra F_σ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then A is a fuzzy supra first category set in (M,S*).

Proof.

Let $A(\neq 1)$ be a fuzzy supra F_σ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.9, A is a fuzzy supra σ-nowhere dense set in (M,S*) and thus A is a fuzzy supra F_σ-set in (M,S*) with $int^*(A)=0$. Then, $1-A$ is a fuzzy supra G_δ-set with $cl^*(1-A)=1-int^*(A)=1-0=1$, in (M,S*). Then, by Theorem 2.4, $1-A$ is a fuzzy supra residual set in (M,S*). Hence A is a fuzzy supra first category set in (M,S*).

The following proposition gives a condition under which fuzzy supra hyperconnected and fuzzy supra almost P-space become fuzzy supra σ-Baire spaces.

Proposition 4.11.

If $int^*[\vee_{i=1}^{\infty}(B_i)]=0$, where (B_i)'s are fuzzy supra F_σ-sets in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then (M,S*) is a fuzzy supra σ-Baire space.

Proof.

Let (A_i)'s be fuzzy supra F_σ-sets in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.9, (A_i)'s are fuzzy supra σ-nowhere dense sets in (M,S*). By hypothesis, $int^*[\vee_{i=1}^{\infty}(A_i)]=0$, where (A_i)'s are fuzzy supra σ-nowhere dense sets in (M,S*). Hence (M,S*) is a fuzzy supra σ-Baire space.

Proposition 4.12.
If $int^*[\vee_{i=1}^{\infty}(B_i)]=0$, where (B_i)'s are fuzzy supra F_σ-sets in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then (M,S*) is a fuzzy supra Voltra space.
Proof.
The proof follows from Proposition 4.11 and Theorem 2.11.

The following proposition gives a condition under which fuzzy supra almost P-spaces become fuzzy supra P-spaces.

Proposition 4.13.
If (M,S*) is a fuzzy supra globally disconnected, fuzzy supra hyperconnected and fuzzy supra almost P-space, then (M,S*) is a fuzzy supra P-space.
Proof.
Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.3, A is a fuzzy supra semi-open set in (M,S*). Also since (M,S*) is a fuzzy supra globally disconnected space, the fuzzy supra semi-open set A is a fuzzy supra open set in (M,S*). Hence (M,S*) is a fuzzy supra P-space.

Proposition 4.14.
If $A(\neq 1)$ is a fuzzy supra σ-boundary set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then

(i) A is a fuzzy supra σ-nowhere dense set in (M,S*).
(ii) A is a fuzzy supra first category set in (M,S*).

Proof.
(i) Let A be a fuzzy supra σ-boundary set in (M,S*). Then, by Theorem 2.2, A is a fuzzy supra F_σ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.9, A is a fuzzy supra σ-nowhere dense set in (M,S*).
(ii). By Theorem 2.2, the fuzzy supra σ-boundary set A is a fuzzy supra F_σ-set in (M,S*). Then, by Proposition 4.10, A is a fuzzy supra first category set in (M,S*).

The following proposition shows that fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra weakly Baire spaces.

Proposition 4.15.
If (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space then (M,S*) is a fuzzy supra weakly Baire space.
Proof.
Let A be a fuzzy supra σ-boundary set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.14 (i), A is a fuzzy supra σ-nowhere dense set in (M,S*). Then, A is a fuzzy supra F_σ-set in (M,S*) with $int^*(A)=0$. Thus, for the fuzzy supra σ-boundary set A in (M,S*), $int^*(A)=0$, implies, by Theorem 2.12, that (M,S*) is a fuzzy supra weakly Baire space.

Proposition 4.16.

If (M,S*) is a fuzzy supra open hereditarily irresolvable, fuzzy supra hyperconnected and fuzzy supra almost P-space, then (M,S*) is a fuzzy supra Baire space.

Proof.

Let (M,S*) be a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.15, (M,S*) is a fuzzy supra weakly Baire space. Thus (M,S*) is a fuzzy supra open hereditarily irresolvable and fuzzy supra weakly Baire space. Then, by Theorem 2.13, (M,S*) is a fuzzy supra Baire space.

Proposition 4.17.

If (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, then A and $1-cl^*(A)$ are disjoint fuzzy supra sets for any fuzzy supra regular open set A in (M,S*).

Proof.

Let A be a fuzzy supra regular open set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.15, (M,S*) is a fuzzy supra weakly Baire space. Then, by Theorem 2.14, for the fuzzy supra regular open set A, $A \wedge int^*(1-A) = 0$, in (M,S*) and then $A \wedge (1-cl^*(A)) = 0$, in (M,S*). Hence A and $1 - cl^*(A)$ are disjoint fuzzy supra sets in (M,S*).

Proposition 4.18.

If A is a fuzzy supra G_δ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then there exists a fuzzy supra F_σ-set B in (M,S*) such that $cl^*(A \vee B)=1$, in (M,S*).

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.2(i), $A \vee (1-A)$ is a fuzzy supra dense set in (M,S*). Since A is a fuzzy supra G_δ-set in (M,S*), $1 - A$ is a fuzzy supra F_σ-set in (M,S*). Let $B= 1-A$ and then B is a fuzzy supra F_σ-set in (M,S*). Then, $A \vee B$ is a fuzzy supra dense set in (M,S*) and hence $cl^*(A \vee B) = 1$, in (M,S*).

Proposition 4.19.

If (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, then (M,S*) is a fuzzy supra irresolvable space.

Proof.

Let $A(\neq 0)$ be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra hyperconnected and fuzzy supra almost P-space, by Proposition 4.2(i), $A \vee (1 - A)$ is a fuzzy supra dense set in (M,S*). Let $C=A \vee (1-A)$. Then, $cl^*(C)=1$, in (M,S*). Since A is a fuzzy supra G_δ-set in (M,S*), $1-A$ is a fuzzy supra F_σ-set in (M,S*). Let $B=1-A$ and $A\neq 0$, implies that $B\neq 1$. Thus $B(\neq 1)$ is a fuzzy supra F_σ-set in (M,S*). Then, by Proposition 4.9, B is a fuzzy supra σ-nowhere dense set in (M,S*) and thus B is a fuzzy supra F_σ-set in (M,S*) with $int^*(B)= 0$, in (M,S*). Now $cl^*(1-C)=cl^*(1-[A \vee (1-A)]) =cl^*(1-[A \vee B])=1-int^*[A \vee B]\leq 1-(int^*[A] \vee int^*[B])= 1-(int^*[A] \vee 0]) =1 -int^*[A]$. Since (M,S*) is a fuzzy supra almost P-space, $int^*(A)\neq 0$ and thus $1 - int^*[A]$ is a non-zero fuzzy supra closed set in (M,S*). Thus, $cl^*(1 - C)\leq 1 - int^*[A]$ implies that $cl^*(1-C)\neq 1$. Hence, for a fuzzy supra dense set C in (M,S*), $cl^*(1-C)\neq 1$, implies that (M,S*) is a fuzzy supra irresolvable space.

Proposition 4.20.

If $A(\neq 1)$ is a fuzzy supra F_σ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then there exists a fuzzy supra F_σ-set B in (M,S*) such that $A \leq B$.

Proof.

Let $A(\neq 1)$ be a fuzzy supra F_σ-set in (M,S*). Since (M,S*) is a hyperconnected and fuzzy supra almost P-space, By Proposition 4.10, A is a fuzzy supra first category set in (M,S*). Then, $A=(\vee_{i=1}^{\infty} C_i)$, where (C_i)'s are fuzzy supra nowhere dense sets in (M,S*). Let $\theta = \wedge_{i=1}^{\infty} [1\text{-}cl^*(C_i)]$. Now $1\text{-}cl^*(C_i)$ is a fuzzy supra open set in (M,S*) and $\theta = \wedge_{i=1}^{\infty}(1\text{-}cl^*(C_i))$ is a fuzzy supra G_δ-set in (M,S*). Also $\theta = \wedge_{i=1}^{\infty}[1\text{-}cl^*(C_i)] = 1\text{-}[\vee_{i=1}^{\infty} cl^*(C_i)] \leq 1\text{-}[\vee_{i=1}^{\infty}(C_i)] = 1\text{-}A$ and then $A \leq 1\text{-}\theta$ in (M,S*). Let $B=1\text{-}\theta$ and thus B is a fuzzy supra F_σ-set, in (M,S*). Hence, for the fuzzy supra F_σ-set A, there exists a fuzzy supra F_σ-set B in (M,S*) such that $A \leq B$.

Proposition 4.21.

If $A(\neq 0)$ is a fuzzy supra G_δ-set in a fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*), then there exists a fuzzy supra G_δ-set B in (M,S*) such that $B \leq A$.

Proof.

Let $A(\neq 0)$ be a fuzzy supra G_δ-set in (M,S*). Then, $1\text{-}A$ is a fuzzy supra F_σ-set in the fuzzy supra hyperconnected and fuzzy supra almost P-space (M,S*). Then, by Proposition 4.20, there exists a fuzzy supra F_σ-set C in (M,S*) such that $1 \text{-}A \leq C$ and $1\text{-} C \leq A$. Let $B=1\text{-}C$. Then, B is a fuzzy supra G_δ-set in (M,S*). Hence, for the fuzzy supra G_δ-set A, there exists a fuzzy supra G_δ-set B in (M,S*) such that $B \leq A$.

Proposition 4.22.

If a fuzzy supra topological space (M,S*) is a fuzzy supra *P*-space, then (M,S*) is a fuzzy supra basically disconnected and fuzzy supra almost *P*-space.

Proof.

Let A be a fuzzy supra G_δ-set in (M,S*). Since (M,S*) is a fuzzy supra *P*-space, A is a fuzzy supra open set and hence $int^*(A) = A \neq 0$, in (M,S*). This implies that (M,S*) is a fuzzy supra almost *P*-space. By Proposition 4.1, (M,S*) is a fuzzy supra basically disconnected space. Hence (M,S*) is a fuzzy supra basically disconnected and fuzzy supra almost *P*-space.

5.Conclusion

In this paper it is established that fuzzy supra G_δ-sets and fuzzy supra residual sets in fuzzy supra almost P-spaces are fuzzy supra somewhere dense sets. It is obtained that fuzzy supra σ-boundary sets in fuzzy supra almost P-spaces are either fuzzy supra first category sets which are not dense or fuzzy supra somewhere dense sets. It is established that fuzzy supra G_δ-sets in fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra dense sets. It is obtained that fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra semi-P-spaces and fuzzy supra weakly Baire spaces. It is established that fuzzy supra basically disconnected and fuzzy supra hyperconnected spaces are not fuzzy supra almost P-spaces. A condition is obtained for fuzzy supra door spaces to become fuzzy supra almost P-spaces by means of somewhere denseness of fuzzy supra closed sets. It is established that fuzzy supra globally disconnected, fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra P-spaces. The conditions under which fuzzy supra almost P-spaces become fuzzy supra Baire spaces, are also obtained. It is obtained that fuzzy supra hyperconnected and fuzzy supra almost P-spaces are fuzzy supra irresolvable spaces.

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Conflict of Interest

The authors declare no conflicts of interest regarding the publication of this article.

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